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ANALYSIS OF WELLBORE CROSSFLOWS FOR NONSTATIONARY OPERATION OF A WELL IN AN INHOMOGENEOUS MULTILAYER BED

K. M. Fedorov,[1] L. M. Kadochnikova,[2] and S. N. Repetov[1]

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This paper considers the nonstationary filtration of a homogeneous elastic fluid in a rigid, inhomogeneous, multilayer bed with no hydrodynamic communication between interbeds. The effect of wellbore crossflow between interbeds after stopping of well operation is studied numerically. It is shown that the volume and time of fluid crossflow between the interbeds are comparable to the volume of injection and the time of chemical treatment of wells. It is concluded that crossflows can be used for selective injection of chemicals. A procedure is proposed to control the volume of selective fluid injection into a well by changing the injection rate.

Vertical fluid crossflows between beds and interbeds play an important role in estimating oil and gas reserves and the efficiency of their production from wells. In the absence of hydrodynamic communication between interbeds, there may be fluid crossflows between the interbeds through production wells at different bed pressures [1, 2]. After a production or injection well that opens several beds (interbeds) stops operating, nonstationary wellbore crossflows are possible even in the case of identical bed pressures. The occurrence of such crossflows after stopping of operation of water wells was proved experimentally and theoretically by Yusupov et al. [3]. Ehlig-Economides and Joseph [4] performed a theoretical analysis of this process in studying the possibility of determining properties of beds and interbeds by well hydrodynamic research. It is shown that simultaneous measurements of pressure at a wellbore and differential discharges in interbeds make it possible to determine the filtration characteristics of a multilayer reservoir.

During operation of oil and gas wells, it is frequently necessary to increase the injectivity of low-permeability zones and interbeds or, on the contrary, to block high-permeability encroached interbeds. For this, the face zones are treated with acids and gels. In these methods, the main problem is related to the selective delivery of chemicals to high- and low-permeability interbeds [5]. In the present paper, we analyze intrareservoir communication through a well that operates in a nonstationary mode. The numerical analysis performed shows that the volume and characteristic time of crossflows with sharp changes in the fluid discharge through the well are comparable to the corresponding quantities obtained for treatment of reservoirs with acids or gels. The effects of the ratio of permeabilities of interbeds and change in fluid discharge on the magnitude of intrareservoir wellbore communication are explored.

1. Formulation of the Problem. We consider the nonstationary operation of a production well which opens an oil bed consisting of several hydrodynamically isolated interbeds with different filtration parameters. The first problem is to analyze fluid flows in the interbeds after stopping of well operation. The process is analyzed within the framework of the model of filtration of a homogeneous elastic fluid in a rigid bed. In this case, the pressure distribution for each interbed is described by the piezoelectric-conductivity equation [6]

$$\frac{\partial P_i}{\partial t} = \chi_i \frac{1}{r} \frac{\partial}{\partial r}\Big(r \frac{\partial P_i}{\partial r}\Big), \tag{1.1}$$

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[1]Tyumen' State University, Tyumen' 625003. [2]Tyumen Branch of the Institute of Theoretical and Applied Mechanics, Siberian Division, Russian Academy of Sciences, Tyumen' 625000. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 42, No. 3, pp. 84–90, May–June, 2001. Original article submitted December 20, 1999; revision submitted October 3, 2000.

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where P is the pressure, $\chi = k/(\mu m \beta)$ is the piezoelectric-conductivity coefficient, m is the porosity of the medium, k is the absolute permeability, μ is the viscosity of the bedded fluid (oil), β is the elastic capacity of an interbed, t is time, and r is the radial coordinate reckoned from the center of the well; the subscript i corresponds to the interbed number.

The initial pressure distribution for the interbeds corresponds to well operation with constant discharge and is defined by the Dupuis formula

$$P_i(r) = P_r + \frac{(P_r - P_w)\ln(r/R)}{\ln(R/r_w)} \qquad (t = 0),$$

where P_r is the pressure on the external boundary of the well (identical for all interbeds), P_w is the bottom-hole pressure (at the bed level), R is the radius of the external boundary of the well, and r_w is the radius of the well. The bed pressure on the external boundary is considered identical for all interbeds: $P_i = P_r$ at $r = R$.

When well operation is stopped, the total discharge of all interbeds is equal to zero. In this case, the bottom-hole pressure for the interbeds is given by the differential relation

$$\sum_i \Big(\frac{2\pi k_i h_i}{\mu_i}\Big(r\,\frac{\partial P_i}{\partial r}\Big)_{r=r_w}\Big) = 0, \tag{1.2}$$

where h is the interbed thickness.

Since the bed thickness is about 10 m, the hydrostatic additive to the wellbore pressure due to the different depth of the interbeds does not exceed 1 atm. The pressure drop in the interbeds produced by pumping equipment is 50–100 atm. Therefore, as a first approximation, it is possible to ignore this additive and assume that wellbore pressure is identical for all interbeds. Allowance for the additive is not difficult and does not change the character of the processes studied. The additional boundary conditions corresponding to stopping of well operation is that the bottom-hole pressure is the same for all interbeds: $P_i = P_j$, where i and j are interbed numbers.

Thus, a feature of this problem is that interrelated values of pressure and their derivatives are assigned on one of the boundaries (on the well wall). In the case of two interbeds, the problem reduces to the classical problem of heat transfer in a composite rod.

The problem was solved by a numerical method. In solving Eq. (1.1) for each interbed, we used an implicit difference scheme of second-order accuracy for the coordinate and first-order accuracy for time. The difference equations were solved by the sweep method. The calculations were carried out on a nonuniform grid with step $dr_j = dr_0 q^j$, where q is the denominator of the geometrical progression and dr_0 is the size of the cell bordering the well. The numerical method was tested by comparison with the well-known analytical solutions. For $dr_0 = 0.01$ m and $q = 1.135$, the error of the pressure distribution and the total fluid discharge was 0.01%.

The problem was solved with allowance for boundary conditions (1.2), which are a system of differential equations that cannot be solved analytically. The algorithm of solution is a modification of the bisection method. For arbitrary time t, the pressure on the boundary identical for all interbeds is chosen. Then, Eq. (1.1) is solved numerically for each interbed, the iterative values of the discharge Q_i^ν for each interbed are determined, and the total fluid delivery to the well is calculated: $Q^\nu = \sum Q_i^\nu$. If the absolute value of the difference between the total flow rate and the well discharge is less than a certain value ε ($|Q^\nu - Q| < \varepsilon$), the obtained pressure distribution in the interbeds is the solution. Otherwise, ($|Q^\nu - Q| > \varepsilon$), if the difference $Q^\nu - Q$ is positive, the iterative value of $P^{\nu+1}$ on the boundary is calculated from the formula $P^{\nu+1} = P^\nu - \Delta P$. If the difference has a negative value, the pressure is calculated as $P^{\nu+1} = P^\nu + \Delta P$. When the discharge Q^ν changes sign, the value ΔP is halved. The proposed algorithm ensures a high rate of convergence (solution is reached in 9–10 iterations) and is implemented in each time step.

2. Calculation Results. Let us consider results of numerical solution of the problem of shutdown of a production well in a bed consisting of two interbeds. In the calculations, the bed pressure was $P_r = 200$ atm. Before $t = 0$, the well operated with a constant discharge of 86.4 m^3/day, and the bottom-hole pressure was 188.92 atm. The interbeds had the following parameters: $h = 10$ m, $m = 0.2$, $\beta = 10^{-9}$ $m \cdot sec^2/kg$, and $\mu = 10^{-3}$ Pa $\cdot$ sec and $k = 10^{-14}$ for the first interbed and 10^{-13} m^2 for the second interbed.

After the production well stops operating, the interbed pressure changes. Figure 1 shows calculation results for part of the external boundary near the well. Initially, the stationary pressure distribution in both interbeds is described by the Dupuis formula (curves 1). In what follows, the pressure distribution in the high-permeability interbed is monotonic (Fig. 1b). In the low-permeability interbed near the well, a sharp pressure drop is observed, and then, as in the high-permeability interbed, the pressure increases monotonically to the bed pressure (Fig. 1a).

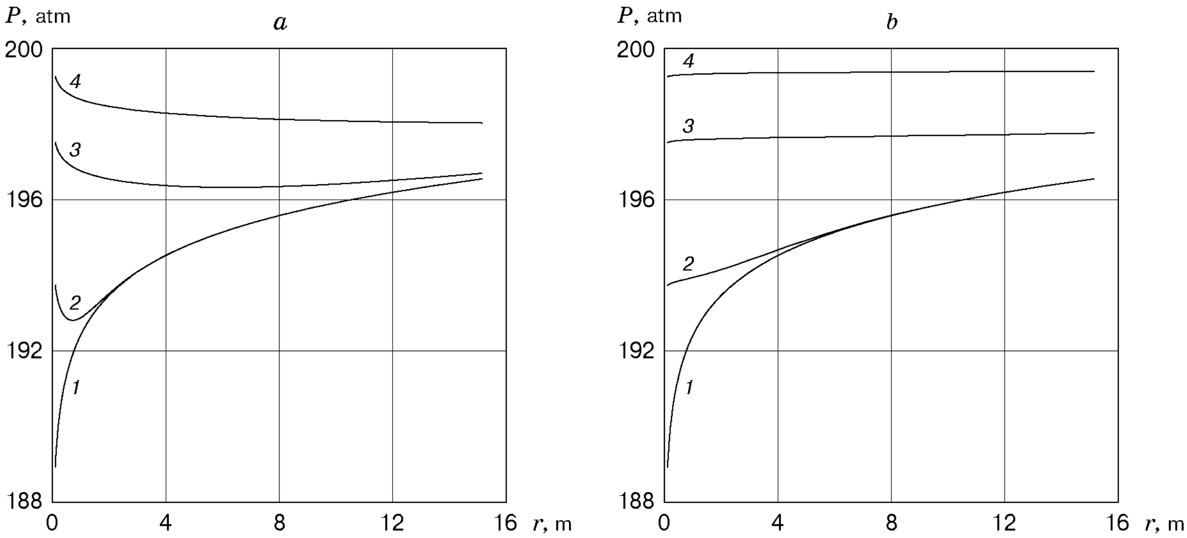

Fig. 1. Pressure distribution in low-permeability (a) and high-permeability (b) interbeds (part of the external boundary near the well is shown) for $t = 0$ (1), 50 (2), 500 (3), and 50,000 sec (4).

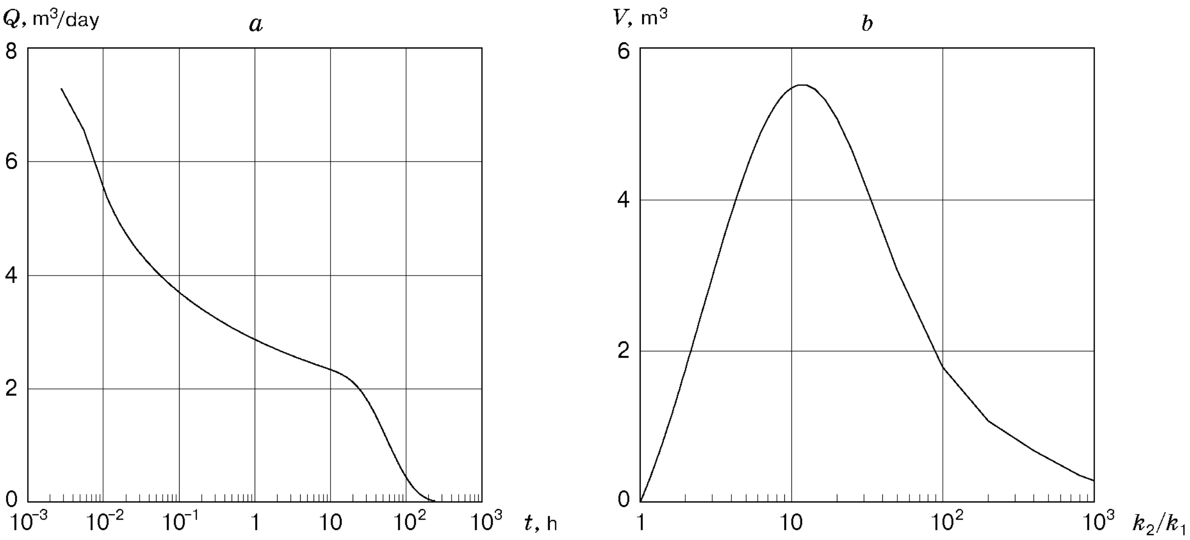

Fig. 2. Fluid crossflow after shutdown of a well that opens a bed consisting of two isolated interbeds: (a) time dependence of the total discharge of the fluid crossflow; (b) dependence of the fluid crossflow volume on the ratio k_2/k_1 at $k_2 = 10^{-13}$ m^2 = const.

With time, the pressure in both interbeds tends to the constant value equal to the bed pressure. Because of the difference in the signs of the pressure gradients on the wellbore wall in the interbeds, there are a decaying fluid flow from the high-permeability interbed and absorption of the fluid from the wellbore by the low-permeability interbed. The fluid volume that has thus flowed between the interbeds depends on both the ratio of the fluid conductivity of the interbeds $(k_i h_i/\mu_i)$ and the absolute values of these coefficients.

Figure 2a gives the time dependence of the total fluid discharge from the high-permeability interbed to the low-permeability bed is introduced. At $t = 0$, the discharge for the problem considered is maximal and corresponds to the well discharge before shutdown. After shutdown, the discharge decreases considerably (by approximately a factor of four in about 20 min). Then, over a long period (tens of hours) pressure relaxation and fluid crossflow are observed.

Figure 2b shows the dependence of the fluid volume that has flowed from the high-permeability to the low-permeability interbed over a time of 48 h on the ratio of permeabilities k_2/k_1 for a fixed value of $k_2 = 10^{-13}$ m^2.

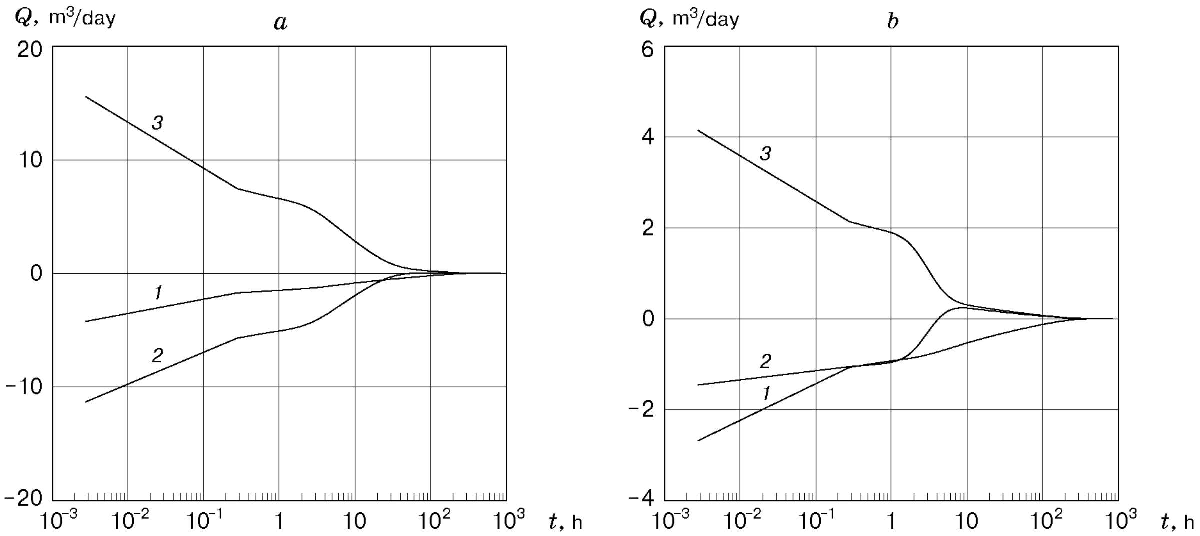

Fig. 3. Time dependence of the fluid crossflow rate in interbeds after shutdown of a well: (a) curve 1 refers $k_1 = 10^{-14}$ m^2 and $V_1 = -2.9$ m^3, curve 2 to $k_2 = 8 \cdot 10^{-14}$ m^2 and $V_2 = -2.4$ m^3, and curve 3 to $k_3 = 10^{-12}$ m^2 and $V_3 = 5.3$ m^3; (b) curve 1 refers to $k_1 = 10^{-14}$ m^2 and $V_1 = -1.8$ m^3, curve 2 to $k_2 = 8 \cdot 10^{-13}$ m^2 and $V_2 = 0.6$ m^3, and curve 3 to $k_3 = 10^{-12}$ m^2 and $V_3 = 1.2$ m^3.

The curve in Fig. 2b has a distinct maximum ($V = 5.5$ m^3) at $k_2/k_1 = 12$. As $k_2/k_1 \to 1$, $V \to 0$ because in a homogeneous bed no fluid crossflow is observed. At $k_2/k_1 > 12$, the fluid crossflow volume decreases, i.e., for $k_2/k_1 \to \infty$, there is no fluid crossflow because the low-permeability interbed becomes impermeable and does not react to a change in the well discharge.

Shutdown of an injection well leads to the opposite effect — fluid flows from the low-permeability interbed and the fluid is absorbed from the well by the high-permeability interbed. For shutdown of an injection well and a production wells, the fluid crossflow rate coincide in magnitude but are opposite in sign (for the same parameters of interbeds and pressure drops). The fluid crossflow volumes for injection and production wells also coincide.

Let us consider numerical results from solution of the problem of shutdown of an injection hole in a bed consisting of three interbeds. The parameters of the interbeds were as follows: $h_1 = h_2 = h_3 = 3$ m, $m_1 = m_2 = m_3 = 0.2$, $\beta_1 = \beta_2 = \beta_3 = 10^{-9}$ m $\cdot$ sec^2/kg, and $\mu_1 = \mu_2 = \mu_3 = 10^{-3}$ Pa $\cdot$ sec; the permeability coefficients were varied. The bed pressure was 150 atm.

After shutdown of the well, a characteristic fluid flow through the bottom hole from the low-permeability to the high-permeability interbeds is observed. The time dependences of the rate of fluid crossflow between the interbeds after shutdown are presented in Fig. 3. The fluid flows from the low-permeability interbeds 1 and 2 and is absorbed from the well by the high-permeability interbed 3 (Fig. 3a). For the conditions corresponding to Fig. 3b, at small values of t, the fluid flows from the low-permeability interbeds 1 and 2 to the high-permeability interbed 3. In about 4 h after shutdown of the well, the direction of the fluid crossflow in the interbed 2 is reversed, and the total fluid volume that has flowed from the interbed 2 is positive. Thus, the fluid absorption from the well after shutdown is a selective process that depends on the permeability of the interbeds.

Results of calculations for the problem of variation in the rate of fluid injection into the well are given below. We considered the process of fluid injection into an injection well that opens a three-layer bed with the parameters described above and permeabilities equal to 10^{-14}, $8 \cdot 10^{-14}$, and 10^{-12} m^2. Figure 4a gives the time dependence of the crossflow rate for the three interbeds with the injection rate varying from 120 to 86.4 m^3/day at time $t = 0$. In the initial period of time (up to $t \approx 0.1$ h) the fluid flows from low-permeability interbeds and is absorbed from the well by the high-permeability interbeds. Then all three interbeds begin to absorb fluid volumes in proportion to their permeabilities. After 10 hours of injection, a stationary distribution of the injected fluid volume in the interbeds is observed.

Figure 4b gives an integral dependence of the fluid crossflow volume for 30 days on the difference of the initial and final fluid discharges ΔQ. The initial discharge was set equal to 252 m^3/day, and the final discharge

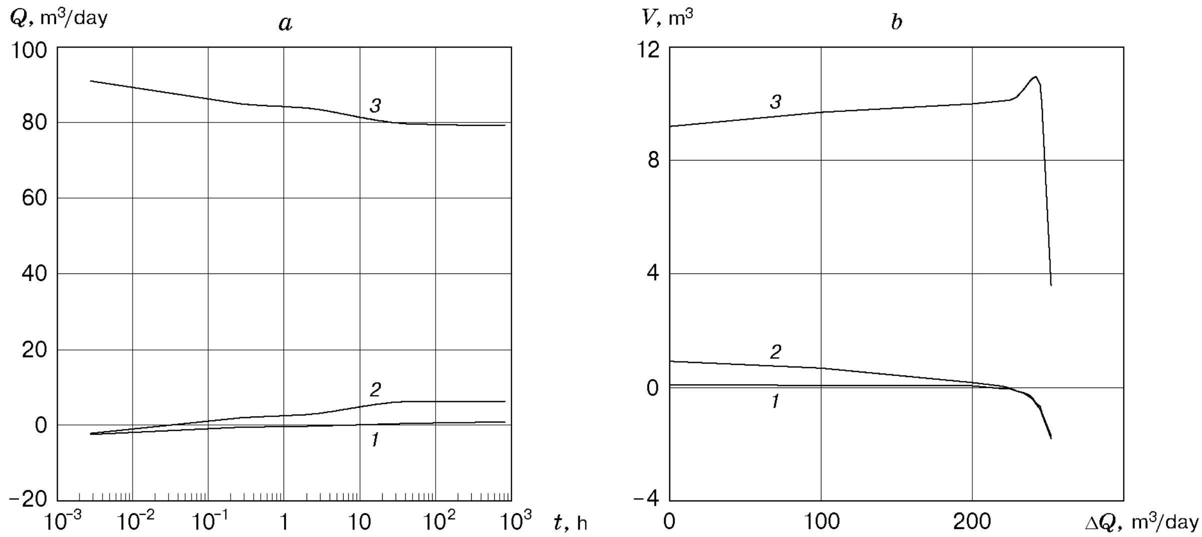

Fig. 4. Fluid crossflow with change of the operation of an injection hole that opens a bed consisting of three isolated interbeds: (a) time dependence of the rate of crossflow between the interbeds with variation in the rate of fluid injection into the bed; (b) dependence of the volume of fluid crossflow between the interbeds on the difference of the initial and final rates of fluid injection into the bed; curves 1 refer to $k_1 = 10^{-14}$ m^2, curves 2 to $k_2 = 8 \cdot 10^{-14}$ m^2, and curves 3 to $k_3 = 10^{-12}$ m^2.

was varied from 252 m^3/day to zero. With a small decrease in the discharge in the layered bed, the fluid volume is distributed in the interbeds in proportion to their fluid conductivities. With larger decrease in discharge, the fluid delivery to the high-permeability interbed increases and the fluid delivery to the low-permeability interbed, vice versa, decreases. For a fluid discharge of more than $\Delta Q = 220$ m^3/day, the entire fluid delivered to the well enters only the high-permeability interbed (selective action on the high-permeability zones of the bed). An analysis of the data in Fig. 4 leads to the conclusion that the fluid volume absorbed by the chosen interbeds can be controlled by varying the flow rate.

Thus, the investigations performed showed that shutdown of a well leads to fluid crossflows between separate beds and interbeds with different filtration characteristics. In this case, the direction of crossflows is determined by the mode of well operation before shutdown: in the case of shutdown of a production well, the fluid flows from the high-permeability interbeds and is absorbed by the low-permeability interbeds, and vice versa, after shutdown of an injection well, the fluid flows from the low-permeability interbeds and is absorbed by the high-permeability beds. The fluid crossflow volume is about 0.5–1.0 m^3 per 1 m of the bed thickness, and the characteristic time is 10–20 h. These data are in good agreement with the corresponding data obtained for chemical treatment of beds.

An analysis of the results of the present work shows that it is possible to use wellbore crossflows for selective delivery of chemicals (for example, gels or acids) to the low or high-permeability parts of beds. The key possibilities of controlling these crossflows are shown.

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